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Potentials of Pointlike Particles in Gauge Theory with a Dilaton

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I examine the potential of a pointlike particle carrying SU (N_c) charge in a gauge theory with a dilaton. The potential depends on boundary conditions imposed on the dilaton: For a dilaton that vanishes at infinity the resulting potential is a regularized Coulomb potential of the form $(r + r_\phi)^{-1}$, with r_ϕ inversely proportional to the decay constant of the dilaton. Another natural constraint on the dilaton ϕ is independence of $(1/g^2)\exp(\phi/f_\phi)$ from the gauge coupling g. This requirement yields a confining potential proportional to r.

1. INTRODUCTION

Dilatons are excitations of a field ϕ which typically couples to other fields χ through terms of the form $\exp(\phi/f_\phi)L(\chi, \partial\chi)$. L can involve curvature terms, Yang–Mills terms, or mass terms, and dilatons arise either as covariant fields under rescalings of four-dimensional coordinates if conformal gravity is constructed with first-order terms in the curvature as in Dirac (1973), or as scalar fields in the framework of string theory and Kaluza–Klein theory (e.g., Green *et al.*, 1987). An unambiguous property of a string dilaton at tree level and a Kaluza–Klein dilaton in four dimensions is, besides its scalar transformation behavior, its coupling to gauge fields through a term $\exp(\phi/f_\phi)F^2$, and we will use this as the defining property of a dilaton. Dilatons arise from the massless spectrum of fundamental strings in two different ways: On the one hand there is the model-independent dilaton arising as a unique massless scalar state of closed superstrings, while on the other hand nonlinear combinations of ten-dimensional massless tensor states measure the volume of internal dimensions and appear as Kaluza–Klein dilatons in four dimensions. Linear combinations of these dilatons may couple in different

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ways to gauge fields arising from different sectors of string theory and from compactifications, and the dilaton sector of low-energy quantum field theories inherited from string theory can become quite complicated. In spite of these possible complications emphasis in the present paper is on the discussion of a single low-energy dilaton coupling to $SU(N_c)$ gauge fields, in order to acquire a better understanding of the impact of dilatonic degrees of freedom in gauge theory.

In spite of the coupling to gauge fields, the coupling of a string dilaton to the Ricci scalar is not Weyl invariant, and the issue arises of whether string theory in the low-energy sector is a Brans–Dicke-type theory with a coupling $\exp(\phi/f'_\phi)R$ or rather predicts Einstein gravity. The latter possibility assumes that the four-dimensional metric is only conformally related to a Kaluza–Klein-type metric arising through compactification from ten to four dimensions. The two alternatives go by the name "string frame" and "Einstein frame," and they are clearly experimentally inequivalent, since, e.g., geodesics and also the evolution of Friedmann cosmologies in the two theories differ considerably. In the present paper I will assume that, at least to first order in the curvature, gravity is described by the Einstein–Hilbert term and not by a Brans–Dicke-type theory. My main justification for this is the observation of Gross and Sloan (1987) that at string tree level no direct coupling of the string dilaton to the Einstein–Hilbert term arises. Furthermore, Damour and Nordtvedt (1993) observed in a very remarkable paper that couplings of the form $U(\phi)R$ evolve to Einstein gravity if $U(\phi)$ has a minimum, and coupling functions of this type may arise through higher loop corrections in string theory.

Besides its appearance in the spectrum of conformal gravity, string theory, and Kaluza–Klein theories, the present investigation was also motivated by the fact that the dilaton begins to play an even more prominent role through its covariance under duality symmetries: It was pointed out in Shapere *et al.* (1991) that axion–dilaton–photon systems exhibit a nonlinear duality symmetry, mixing axions and dilatons through SL(2, R) transformations, and it has been conjectured that S-duality symmetries should be a generic feature of the kinetic sector of low-energy quantum field theories (e.g., Font *et al.*, 1990; Schwarz and Sen, 1994).

It is familiar from the axion that a (pseudo-scalar) can be very light, yet very hard to observe if its decay constant is large enough to suppress the couplings at low energies. It is apparent from the equations of motion considered below that the same assertion holds for the dilaton. A relevant problem then is the question of how a light dilaton affects the Coulomb potential and its non-Abelian analog. In order to study this problem we will neglect any curvature effects and dilaton mass in the sequel and study the dilaton–gluon field generated by a pointlike quark in Minkowski space. For a dilaton vanishing at infinity we will then find a modified Coulomb potential which is regularized at a radius

$$r_\phi = \frac{g}{8\pi f_\phi}\sqrt{\frac{1}{2} - \frac{1}{2N_c}}$$

for gauge group $SU(N_c)$ and

$$r_\phi = \frac{g}{8\pi f_\phi}$$

for U(1).

On the other hand, a nonvanishing expectation value of the dilaton can be absorbed in a rescaling of the gauge coupling, and this gives rise to the requirement that $\exp(\phi/f_\phi)$ should scale like g^2. The unique solution satisfying this requirement yields a potential proportional to the distance r from the source.

Throughout this paper the language of QCD will be used for gauge fields, charges, and fermions. Letters from the middle of the alphabet will be used both for spatial Minkowski space indices and for color indices, while letters from the beginning of the alphabet denote Lie algebra indices. I also use the letter Φ for the first component A^0 of the gauge potential, and the signature of the metric is $(-+++)$.

2. THE GENERALIZED COULOMB POTENTIALS

As pointed out in the previous section, we expect dilatonic degrees of freedom in four-dimensional gauge theories if physics at very high energies involves decompactification of internal dimensions or string theory. To acquire a better understanding of the impact of dilatons in four-dimensional gauge theory, we now will look into the problem of how a light dilaton modifies the Coulomb potential and its non-Abelian analog. It turns out that the dilaton introduces an ambiguity due to different boundary conditions which can be imposed on the dilaton: Two interesting solutions which arise include a regularized potential proportional to $(r + r_\phi)^{-1}$, where r_ϕ is inversely proportional to the decay constant of the dilaton, and a confining potential proportional to r.

Here we are interested in low-energy gauge theories, i.e., in the dynamics of initially massless modes from the point of view of string theory. Since the compactification scale or string scale is many orders of magnitude larger than the weak scale, where the low-energy degrees of freedom described in the standard model of particle physics acquire their masses, we do not expect the dilaton to couple to the relevant masses at the weak scale. Modulo an effective potential which the dilaton may have acquired on the road down from the string/compactification scales to temperatures below the SUSY scale, the influence of a dilaton on a low-energy gauge theory is then described by a Lagrange density

$$\mathcal{L} = -\frac{1}{4}\exp\left(\frac{\phi}{f_\phi}\right)F^a_{\mu\nu}F^{\mu\nu}_a - \frac{1}{2}\partial^\mu\phi \cdot \partial_\mu\phi + \sum_{f=1}^{N_f}\bar{\psi}_f(i\gamma^\mu\partial_\mu + g\gamma^\mu A^a_\mu X_a - m_f)\psi_f \tag{1}$$

with X_a denoting a defining N_c-dimensional representation of su(N_c).

A Lagrangian of this type would arise, e.g., through compactification of five-dimensional QCD (Dick, 1996), with exception of the mass terms, which are assumed to arise at a lower scale. Axionic contributions have been neglected, since the static pointlike sources considered below would not excite the axion field.

The equations of motion are

$$\partial_\mu\left(\exp\left(\frac{\phi}{f_\phi}\right)F^{\mu\nu}_a\right) + g\exp\left(\frac{\phi}{f_\phi}\right)A^b_\mu f^c_{ab}F^{\mu\nu}_c = -g\bar{q}\gamma^\nu X_a q \tag{2}$$

$$\partial^2\phi = \frac{1}{4f_\phi}\exp\left(\frac{\phi}{f_\phi}\right)F^a_{\mu\nu}F^{\mu\nu}_a \tag{3}$$

$$(i\gamma^\mu\partial_\mu + g\gamma^\mu A^a_\mu X_a - m)q = 0 \tag{4}$$

where here and in the sequel flavor indices are suppressed. To analyze equation (2) we will find it convenient to rewrite it in terms of the chromoelectric and magnetic fields $E_i = -F^a_{0i}X_a$, $B^i = \frac{1}{2}\epsilon^{ijk}F^a_{jk}X_a$:

$$\nabla \cdot \left(\exp\left(\frac{\phi}{f_\phi}\right)\mathbf{E}\right) - ig\exp\left(\frac{\phi}{f_\phi}\right)(\mathbf{A}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{A}) = \rho$$

$$\partial_0\left(\exp\left(\frac{\phi}{f_\phi}\right)\mathbf{E}\right) - \nabla \times \left(\exp\left(\frac{\phi}{f_\phi}\right)\mathbf{B}\right) + ig\exp\left(\frac{\phi}{f_\phi}\right)([\Phi, \mathbf{E}] + \mathbf{A}\times\mathbf{B} + \mathbf{B}\times\mathbf{A}) = -\mathbf{j}$$

$$\partial_0\mathbf{B} + ig[\Phi, \mathbf{B}] + \nabla\times\mathbf{E} - ig(\mathbf{A}\times\mathbf{E} + \mathbf{E}\times\mathbf{A}) = 0$$

$$\nabla\cdot\mathbf{B} - ig(\mathbf{A}\cdot\mathbf{B} - \mathbf{B}\cdot\mathbf{A}) = 0$$

where in the gauge theory above $\rho = g(q^+ \cdot X_a \cdot q)X^a$, $j_i = g(\bar{q}\cdot\gamma_i X_a \cdot q)X^a$, and we have included the Bianchi identities.

To discuss the impact of the dilaton on the Coulomb potential we consider static configurations: $\partial_0\rho = 0, \mathbf{j} = 0$. Then we learn from $\partial_\mu j^\mu - ig[A_\mu, j^\mu] = 0$ that Φ and ρ are in the same Cartan subalgebra: $[\Phi, \rho] = 0$.

Pointlike stationary charge distributions, which in the present setting give rise to the generalized Coulomb potentials, are special cases of $SU(N_c)$ currents of the form

$$j^\mu(x) = \rho^a(\mathbf{r})X_a\eta_0^\mu = \rho(\mathbf{r})C^aX_a\eta_0^\mu \tag{5}$$

carrying the same **r** dependence along any direction in color space. Such distributions arise for separable quark wave functions $q(x) = \psi(x)\zeta$, where ζ is a constant Lorentz scalar in a spinor representation of $SU(N_c)$, and $\psi(x)$ is an $SU(N_c)$-invariant Dirac spinor whose left- and right-handed components differ only by a phase. We also assume both factors normalized according to $\int d^3\mathbf{r}\bar{\psi} \cdot \psi = 1, \zeta^+ \cdot \zeta = 1$.

For $SU(N_c)$ charges of the form (5) the vector potential can consistently be neglected, whence $\mathbf{E} = -\nabla\Phi$ and the Yang–Mills equations reduce to

$$\nabla \cdot \left(\exp\left(\frac{\phi}{f_\phi}\right)\nabla\Phi\right) = -\rho$$

$$[\Phi, \nabla\Phi] = 0$$

Due to (5) the second equation is fulfilled as a consequence of the first equation.

Our aim is to determine the chromo-electric potential for a point charge

$$\rho_a(\mathbf{r}) = gC_a\delta(\mathbf{r})$$

where C_a denotes the expectation value of the generator X_a in color space. From the relation

$$(X_a)_{ij}(X^a)_{kl} = \frac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{2N_c}\delta_{ij}\delta_{kl} \tag{6}$$

one finds for arbitrary color content

$$\sum_{\alpha=1}^{N_c^2-1} C_a^2 = \frac{N_c - 1}{2N_c}$$

We thus want to determine the field of a stationary pointlike quark from

$$\nabla \cdot \left(\exp\left(\frac{\phi(\mathbf{r})}{f_\phi}\right)\mathbf{E}_a(\mathbf{r})\right) = gC_a\delta(\mathbf{r}) \tag{7}$$

$$\nabla \times \mathbf{E}_a(\mathbf{r}) = 0 \tag{8}$$

and

$$\nabla\phi(\mathbf{r}) = -\frac{1}{2f_\phi}\exp\left(\frac{\phi(\mathbf{r})}{f_\phi}\right)\mathbf{E}_a(\mathbf{r})\cdot\mathbf{E}^a(\mathbf{r}) \quad (9)$$

The unique radially symmetric solution to (7) can be written down immediately:

$$\exp\left(\frac{\phi(r)}{f_\phi}\right)\mathbf{E}_a(\mathbf{r}) = \exp\left(\frac{\phi(r)}{f_\phi}\right)E_a(r)\mathbf{e}_r = \frac{gC_i}{4\pi r^2}\,\mathbf{e}_r \quad (10)$$

whence equation (8) is also satisfied. Equation (9) then translates into

$$\frac{d^2}{dr^2}\phi(r) + \frac{2}{r}\frac{d}{dr}\phi(r) = -\frac{g^2}{64\pi^2 f_\phi}\left(1-\frac{1}{N_c}\right)\exp\left(-\frac{\phi(r)}{f_\phi}\right)\frac{1}{r^4} \quad (11)$$

The form of this equation suggests an ansatz $\phi(r)/f_\phi = a\ln(r/b)$, which yields the solution discussed below. However, we can solve (11) for arbitrary boundary conditions through a substitution

$$\xi = \frac{g}{4\pi f_\phi r}\sqrt{\frac{1}{2}-\frac{1}{2N_c}}, \qquad \varphi(\xi) = \frac{\phi(r)}{f_\phi} \quad (12)$$

yielding[2]

$$\frac{d^2}{d\xi^2}\varphi(\xi) = -\frac{1}{2}\exp(-\varphi(\xi)) \quad (13)$$

or in terms of boundary conditions at infinity:

$$\varphi'(\xi)^2 - \varphi'(0)^2 = \exp(-\varphi(\xi)) - \exp(-\varphi(0)) \quad (14)$$

$$\xi = \int_{\varphi(0)}^{\varphi(\xi)}\frac{d\varphi}{\sqrt{\exp(-\varphi)-\exp(-\varphi(0))+\varphi'(0)^2}}$$

where a sign ambiguity has been resolved by the requirement that the dilaton

[2] We can map the dilaton equation of motion for arbitrary number d of spatial dimensions to equation (13) through the substitution

$$\xi = \frac{g}{f_\phi}\sqrt{\frac{1}{2}-\frac{1}{2N_c}}\,G_d(r)$$

with

$$G_d(r) = -\frac{1}{2\pi}\ln\left(\frac{r}{r_0}\right), \qquad d = 2$$

$$G_d(r) = \frac{\Gamma(d/2)}{2(d-2)\sqrt{\pi}^d}\frac{1}{r^{d-2}}, \qquad d > 2$$

should not diverge at finite radius. The integral can be done in an elementary way, with two branches depending on the sign of $\varphi'(0)^2 - \exp(-\varphi(0))$.

The presence of the dilaton introduced a twofold ambiguity in the Coulomb problem, and we have to determine from physical requirements which boundary conditions to choose.

For a first solution we require that the dilaton generated by the pointlike quark vanishes at infinity while the gradient satisfies the minimality condition

$$\lim_{r\to\infty} r^2 \frac{d}{dr}\phi(r) = -\frac{g}{4\pi}\sqrt{\frac{1}{2} - \frac{1}{2N_c}} \tag{15}$$

This gives minimal kinetic energy for the dilaton at infinity subject to the constraint that the chromo-electric field does not develop a singularity for positive finite r. Then we find for the radial dependence of the dilaton and the electric field

$$\phi(r) = 2f_\phi \ln\left(1 + \frac{g}{8\pi f_\phi r}\sqrt{\frac{1}{2} - \frac{1}{2N_c}}\right) \tag{16}$$

$$\mathbf{E}_a(\mathbf{r}) = \frac{gC_a}{4\pi(r + (g/8\pi f_\phi)\sqrt{1/2 - 1/(2N_c)})^2}\,\mathbf{e}_r \tag{17}$$

implying a modified Coulomb potential

$$\Phi_a(r) = \frac{gC_a}{4\pi r + (g/2f_\phi)\sqrt{1/2 - 1/(2N_c)}} \tag{18}$$

The result for gauge group U(1) is obtained through the substitution $N_c \to -1$, and the corresponding dilaton–photon configuration was proposed already as a solitonic solution in a remarkable paper by Cvetič and Tseytlin (1994).

The removal of the short-distance singularity in the chromo-electric field would imply finite energy of the dilaton–gluon configuration:

$$\begin{aligned} E &= \int d^3\mathbf{r}\left(\frac{1}{2}\nabla\phi\cdot\nabla\phi + \frac{1}{2}\exp\left(\frac{\phi(\mathbf{r})}{f_\phi}\right)\mathbf{E}_a(\mathbf{r})\cdot\mathbf{E}^a(\mathbf{r})\right) \\ &= 2gf_\phi\sqrt{\frac{1}{2} - \frac{1}{2N_c}} \end{aligned} \tag{19}$$

However, there exists another quite intriguing solution if we require that $(1/g^2)\exp(\phi/f_\phi)$ is independent of g. This requirement arises naturally in string theory, since the nonperturbatively fixed expectation value of the string dilaton is supposed to determine the coupling. In the action (1) this requirement amounts to the constraint that the solution should respect the scale invariance of the equations of motion under

$$\phi \rightarrow \phi + 2\eta f_\phi$$
$$A \rightarrow \exp(-\eta)A$$
$$g \rightarrow \exp(\eta)g$$

for constant η. Equations (12) and (14) then imply $\varphi'(\xi)^2 = \exp(-\varphi(\xi)) = 4\xi^{-2}$, yielding

$$\phi(r) = 2f_\phi \ln\left(\frac{g}{8\pi f_\phi r}\sqrt{\frac{1}{2} - \frac{1}{2N_c}}\right) \tag{20}$$

$$\mathbf{E}_a(\mathbf{r}) = \frac{32\pi f_\phi^2}{g}\frac{N_c}{N_c - 1}C_a\mathbf{e}_r \tag{21}$$

This corresponds to an energy density

$$\mathcal{H}(\mathbf{r}) = 4\frac{f_\phi^2}{r^2}$$

whence the energy in a volume of radius r diverges linearly:

$$E|_r = 16\pi f_\phi^2 r$$

This is an infrared divergence which is not related to new physics at short distances, and it would cost an infinite amount of energy to create an isolated quark. Therefore it is very tempting to conclude that a dilatonic degree of freedom is responsible for confinement in QCD.

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